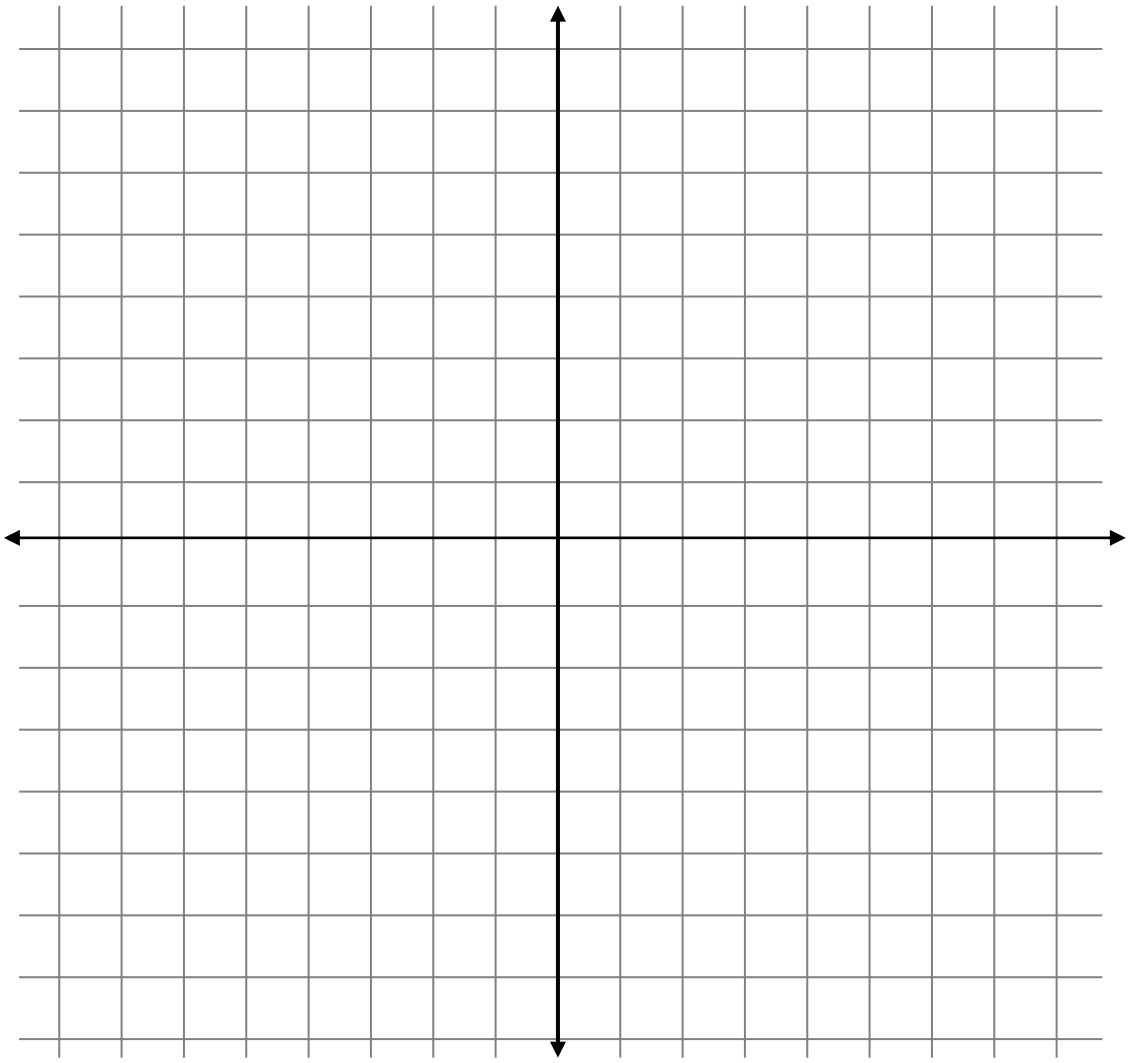
Graphing Rational Functions

Parent Graph of Rational Functions: $f(x) = \frac{1}{x}$

Fill in the table of values and graph:

x	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x)$															



Vertical Asymptote: _____

Horizontal Asymptote: _____

Horizontal Asymptote gives _____.

$As\ x \to \infty, y \to$ _____

$As\ x \to -\infty, y \to$ _____

Graphing Rational Functions

In order to graph rational functions it is useful to determine the following:

1. _____ 4. _____

2. _____ 5. _____

3. _____

Intercepts

1. To find the y-intercept _____

2. To find the x-intercept(s) _____

Examples: Find the x and y intercepts.

1. $f(x) = \frac{(x+1)(x+6)}{(x+2)}$ 2. $g(x) = \frac{-x+2}{(x+1)(x+5)}$ 3. $h(x) = \frac{(x^2-1)}{(x-3)(x+2)}$

End Behavior

1. The end behavior of polynomials was dictated by _____.

2. This is true of rational functions also. To determine end behavior look at the _____ of the numerator and the _____ of the denominator.

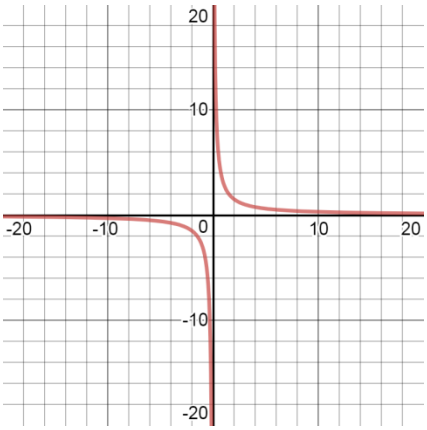
3. To understand end behavior of rational functions fill in the tables below.

x	10	100	1000	10,000	100,000	$As\ x \to \infty$
$y = \frac{1}{x}$						$y \to$

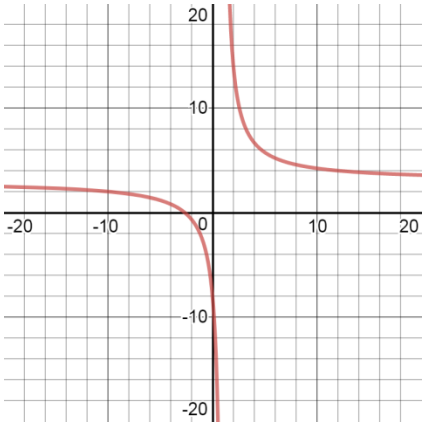
x	-10	-100	-1000	-10,000	-100,000	$As\ x \to -\infty$
$y = \frac{1}{x}$						$y \to$

4. Examine the following graphs to see the 3 kinds of end behavior and make a conjecture that connects the end behavior to the function equation.

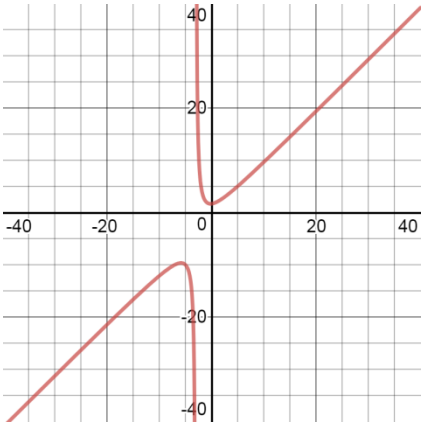
a) $y = \frac{3x}{x^2}$



b) $y = \frac{3x+8}{x-1}$



c) $y = \frac{x^2+2x+5}{x+3}$



Steps for End Behavior (Horizontal Asymptotes)

1. _____

2. _____

3. _____

4. _____

Examples: Determine the end behavior.

1. $f(x) = \frac{(x-1)(x+2)}{x}$

2. $g(x) = \frac{2x+1}{x^2}$

3. $h(x) = \frac{2x^2-3x}{x^2+2}$

Intercepts and End Behavior of Rational Functions

Find the x and y intercepts and the end behavior of each function.

Function	y-intercept	x-intercept	End Behavior
1. $\frac{(x+2)(x-3)}{x}$			$As\ x \to \infty, y \to \_\_\_\_$ $As\ x \to -\infty, y \to \_\_\_\_$
2. $\frac{3}{x^2+2x-3}$			$As\ x \to \infty, y \to \_\_\_\_$ $As\ x \to -\infty, y \to \_\_\_\_$
3. $\frac{x^2+7x+12}{x^3+2x}$			$As\ x \to \infty, y \to \_\_\_\_$ $As\ x \to -\infty, y \to \_\_\_\_$
4. $\frac{(x-1)(x+5)}{(x-2)(x+3)}$			$As\ x \to \infty, y \to \_\_\_\_$ $As\ x \to -\infty, y \to \_\_\_\_$
5. $\frac{x^2-4x+4}{x^2+5x+6}$			$As\ x \to \infty, y \to \_\_\_\_$ $As\ x \to -\infty, y \to \_\_\_\_$
6. $\frac{(x+7)(x-5)}{3}$			$As\ x \to \infty, y \to \_\_\_\_$ $As\ x \to -\infty, y \to \_\_\_\_$
7. $\frac{x^3+3x^2}{x^2+2x-3}$			$As\ x \to \infty, y \to \_\_\_\_$ $As\ x \to -\infty, y \to \_\_\_\_$

Asymptotes of Rational Functions

Find the asymptotes and holes of each function.

Function	Vertical Asymptotes	Horizontal Asymptote
1. $f(x)=\frac{2x}{3x^2+1}$		
2. $f(x)=\frac{2x^2}{x^2-1}$		
3. $f(x)=\frac{5x^2}{x^2-1}$		
4. $f(x)=\frac{x^2+6x+9}{x^2+4x+3}$		
5. $f(x)=\frac{3x^2+1}{x}$		
6. $f(x)=\frac{(x+1)(x-3)}{x^3}$		
7. $f(x)=\frac{x^3+3x^2}{x^2+2x-3}$		

Intercepts, End Behavior, and Sign Charts of Rational Functions

Find the x and y intercepts and the end behavior of each function.

Function	y-intercept	x-intercept	End Behavior	Sign Chart
1. $f(x)=\frac{2x}{3x^2+1}$			$As\ x\to\infty, y\to$ ____ $As\ x\to-\infty, y\to$ ____	
2. $f(x)=\frac{2x^2}{x^2-1}$			$As\ x\to\infty, y\to$ ____ $As\ x\to-\infty, y\to$ ____	
3. $f(x)=\frac{5x^2}{x^2-1}$			$As\ x\to\infty, y\to$ ____ $As\ x\to-\infty, y\to$ ____	
4. $f(x)=\frac{x^2+6x+9}{x^2+4x+3}$			$As\ x\to\infty, y\to$ ____ $As\ x\to-\infty, y\to$ ____	
5. $f(x)=\frac{3x^2+1}{x}$			$As\ x\to\infty, y\to$ ____ $As\ x\to-\infty, y\to$ ____	
6. $f(x)=\frac{(x+1)(x-3)}{x^3}$			$As\ x\to\infty, y\to$ ____ $As\ x\to-\infty, y\to$ ____	
7. $f(x)=\frac{x^3+3x^2}{x^2+2x-3}$			$As\ x\to\infty, y\to$ ____ $As\ x\to-\infty, y\to$ ____	

Identify the features of the function, complete the sign line, and then sketch the function.

4. $f(x) = \frac{2x}{x-3}$

Domain:

Intercepts:

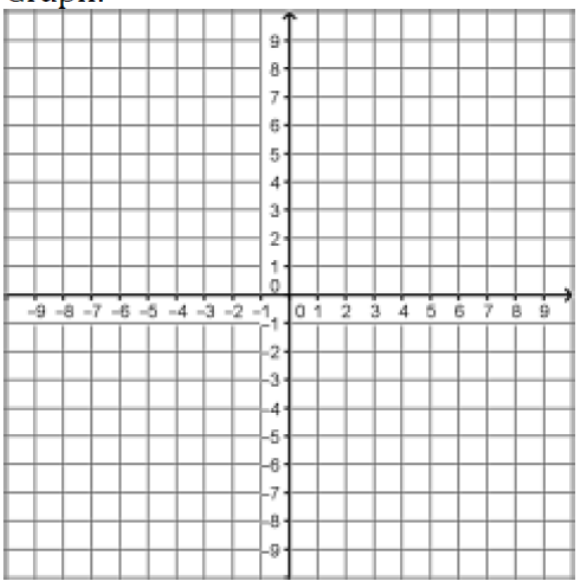
End Behavior:

Vertical Asymptote(s):

Horizontal Asymptote:

Sign Line:

Graph:



5. $g(x) = \frac{1}{(x-2)(x+2)}$

Domain:

Intercepts:

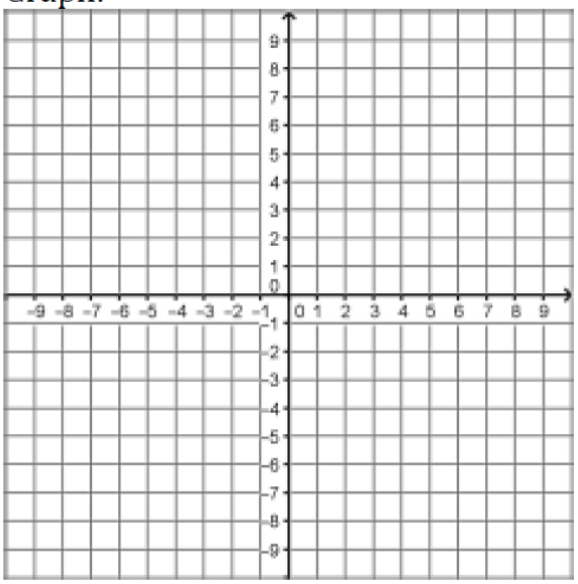
End Behavior:

Vertical Asymptote(s):

Horizontal Asymptote:

Sign Line:

Graph:



6. $h(x) = \frac{x-2}{x-1}$

Domain:

Intercepts:

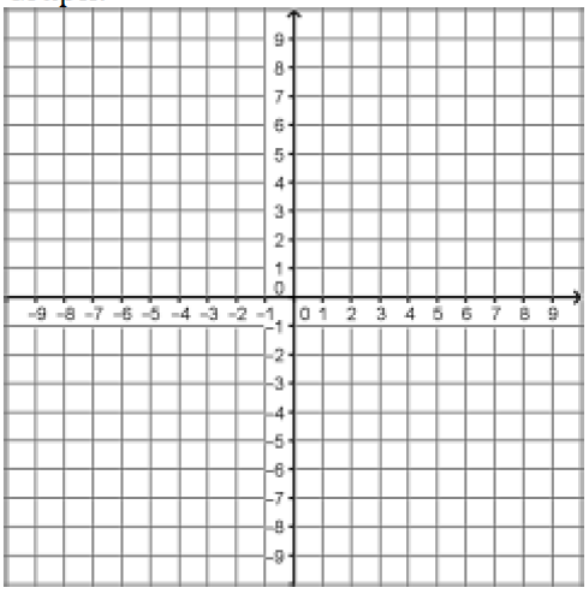
End Behavior:

Vertical Asymptote(s):

Horizontal Asymptote:

Sign Line:

Graph:



7. $w(x) = -\frac{1}{x^2}$

Domain:

Intercepts:

End Behavior:

Vertical Asymptote(s):

Horizontal Asymptote:

Sign Line:

Graph: